Using Profit Maximizing Scheduling Models to Structure Operational Trade-Offs and Manufacturing Strategy Issues

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Abstract

Manufacturing plays an increasingly important role in determining the competitiveness of the firm. However, corporate strategy is often formulated with little regard for how these decisions affect operations within the manufacturing system. Detailed models provide a necessary link between manufacturing performance and the functional policies followed by the firm, so that the strengths of the manufacturing system can be consistently reflected in strategic decisions.

This paper presents a scheduling model that relates the strategic decisions that determine the type of work that must ultimately be processed by the manufacturing system with the detailed decisions that determine how this work should be scheduled. The model accounts for

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varying processing time, delay penalty, and revenue characteristics among the jobs available for processing by a single facility, with jobs partitioned in multiple classes such that a setup is incurred each time two jobs of different classes are processed in succession. Given limited processing capacity, the objective is to simultaneously determine the subset of jobs to accept for processing and the associated order in which accepted jobs should be sequenced to maximize the total profit realized by the facility. Problem formulations and dynamic programming algorithms are presented for both the special case in which all available work is from a single job class, and the more general case involving multiple job classes. The insight derived from these models concerning the operational implications of strategic decisions is illustrated through a series of example problems, first focusing on the coordination of marketing and manufacturing policy, and finally by considering important issues related to manufacturing focus.

1. Introduction

Many organizations have discovered the critical role that manufacturing plays in determining the competitive position of the firm. In sharp contrast with the traditional view of manufacturing as a tactical, reactive, cost-minimizing function, effective design and management of the production system is now considered a sustainable source of competitive advantage (see, *e.g.*, Skinner 1978, 1985, Hayes and Wheelwright 1984, and Hill 1989).

In the past decade, the research literature has responded enthusiastically to the notion that manufacturing should be managed as a strategic asset. Most of this research effort has concentrated on defining the strategic dimensions of manufacturing, and developing the basic concepts that guide decision makers in formulating manufacturing strategy that is consistent with overall corporate strategy (see, *e.g.*, the reviews of St. John 1986, Adam and Swamidass 1989, and Anderson *et al.* 1989). Though the existing literature has clearly established the need to make operational trade-offs in strategic decision making (*e.g.*, to emphasize one manufacturing objective over others), and while some of the most powerful conceptual lessons from manufacturing strategy implicitly impose trade-offs (*e.g.*, focusing a facility often forces hard choices between product line diversity and efficient utilization of productive resources), strategy research has made little use of economic paradigms and operations research models to structure and quantify these trade-offs. Reviews of models that detail the relationship between various strategic manufacturing decisions and the associated tactical challenges placed on operations are provided in Cohen and Lee (1985), Eliashberg and Steinberg (1991), and Fine (1993).

The general types of operations research models used to link strategic and detailed decisions include (*i*) simple inventory models which capture setup cost, inventory holding cost, and pricing considerations (Porteus 1985 and DeGroote 1991), (*ii*) aggregate planning models that account for the interaction between pricing and advertising decisions and short-term capacity strategies for accomodating varying demand (see, *e.g.*, Welam 1977), (*iii*) optimal control theoretic models of timing decisions concerning promotion, production, or new product development (Sogomonian and Tang 1990 and Cohen and Eliashberg 1991), and (*iv*) game theoretic models that address coordination strategies and incentive schemes among marketing and manufacturing managers to improve overall profitability (see, *e.g.*, Jorgensen 1986 and Porteus and Whang 1991). Notably absent from this list are any models linking strategic decisions that affect the character of work that ultimately must be processed by the manufacturing system with detailed decisions that determine how this work should be scheduled. In the final analysis, it is at this most detailed, shop floor level that the implications of these strategic choices are most keenly felt. For example, the choice of processing technology affects manufacturing lead times, setup times, and total capacity; similarly, marketing decisions determine total product volume and mix, thus affecting the potential utilization of facility resources. Detailed models of the scheduling environment offer effective support for strategic decision making by allowing the operational implications of strategic choice to be explicitly and rigorously considered.

The vast majority of research on sequencing and scheduling has assumed that the set of work that must be processed by a facility is given, and that objectives related to minimizing cost, maximizing throughput, or minimizing delay penalties incurred for late deliveries are sufficient for evaluating the quality of scheduling decisions (see Graves 1981, Blazewicz 1987, and Lawler *et al.* 1993). Typically, these studies provide algorithmic and/or heuristic solution procedures for determining the order in which work should be sequenced to meet the stated objective. Such an approach is entirely consistent with the reactive role commonly assumed by manufacturing in corporate strategy.

For manufacturing to be an equal partner in the formulation of corporate strategy, a new class of detailed models are needed to clearly communicate the impact of other functional policies on the operation of the manufacturing system. Only then can firms overcome the tendency to impose production requirements on manufacturing systems that cannot be profitably met, and move toward policies where only the work that the current manufacturing infrastructure can best support is accepted for processing. These models must reflect not only traditional manufacturing objectives, but also market related characteristics such as price and customer reaction to late deliveries. The models should include sufficient detail to accurately capture the operational trade-offs that are germane to the development of solid manufacturing strategy, while at the same time avoiding inessential complexity.

In this paper, we present a scheduling model that accounts for the different processing time, delay penalty, and revenue characteristics associated with a set of jobs available for processing by a single facility. The set of jobs is partitioned according to job class such that if two jobs of different classes are processed in succession, a setup is incurred between the jobs. The objective is to simultaneously determine the subset of jobs to accept for processing, given job class dependent setup times and tight capacity constraints, and the associated order in which the accepted jobs should be sequenced to maximize the total profit realized by the facility. The problem is defined and formulated in Section 2 for both the special case where all available work is of a single job class, and the more general case involving multiple job classes. Dynamic programming solution approaches for the single and multiple job class scheduling problems are presented in Section 3. These algorithms provide valuable decision support by quantifying the impact on profitability of varying the operating environment (*e.g.*, setup times, processing times, delay penalties, or sequencing rules) faced by the manufacturing system, and by structuring the operational trade-offs associated with specific marketing strategies (*e.g.*, product focused policies targeting specific market segments or standardization/customization strategies). Example problems are provided in Section 4 to illustrate how the models can be used to address strategic issues affecting the marketing/manufacturing interface. Similar problems highlighting the operational impact of strategies concerning manufacturing focus are presented in Section 5. Section 6 concludes with a summary.

2. The Model

Consider a single facility capable of processing multiple batches of related jobs. Let $M = \{1, 2, ..., m\}$ denote the set of job classes, and $J = \{1, 2, ..., n\}$ the set of available jobs. The job class of job $j \in J$ is represented by $y(j)$, with $i \sim j$ indicating that jobs i and j are of the same class, *i.e.*, $y(i) = y(j)$. Setup requirements for the facility are such that if two jobs of the same class are processed in succession, no setup is incurred between the jobs; conversely, a known setup time is required whenever two jobs of different types are processed consecutively. Thus, if h_k represents the setup time incurred prior to the production of a batch of job class k, and job j is of type k (*i.e.*, $y(j) = k$), then no setup time is incurred between the processing of consecutive jobs i and j when $i \sim j$, while h_k units of setup time are required when i and j are of different job classes. This structure of setup times is a special case of sequence-dependent setups, where both preceding and succeeding jobs affect setup times.

Associated with each job $j \in J$ is a processing time t_j. For a given schedule, job j completes at time C_j, generating $R_j(C_j)$ units of revenue. A revenue function that is decreasing in completion times is consistent with a time-competitive production environment; for modeling simplicity we hereafter assume the following linear form:

$$R_j(C_j) = R_j^0 - a_j C_j, \tag{1}$$

where a_j denotes the rate at which revenue decreases as the completion time of job j is delayed. $R_j(C_j)$ can also be interpreted as a profit function, with R_j^0 representing the price quoted to the customer demanding job j and $a_j C_j$ the processing (*e.g.*, inventory holding) cost of job j. If the total amount of available work exceeds the processing capacity of the facility over a finite planning horizon T, then both the subset of jobs in set J to accept for processing and the sequence in which the accepted jobs should be completed must be determined to maximize total profit, defined as total revenue net of delay penalties or manufacturing costs.

Setup times that depend only on the job classes of successive jobs have been previously investigated in the scheduling literature (*e.g.*, see the review of Webster and Baker 1995). Bruno and Downey (1978) consider a single-machine scheduling problem with deadlines and job type dependent setups, with emphasis placed on determining if there exists a production schedule that meets all job due dates. The problem is shown to be NP-complete, and a pseudo-polynomial solution approach (exponential in the number of distinct due dates) is provided. Monma and Potts (1989) present an alternative algorithm for the problem which is exponential in the number of distinct job classes. Bruno and Sethi (1978) address the single-machine problem with job type dependent setup times and a total weighted flow time scheduling criterion, and suggest a dynamic programming approach for the problem. Monma and Potts (1989) also present a dynamic programming solution procedure for this problem that is polynomial in the number of jobs, but exponential in the number of job classes. An algorithm developed by Gupta (1984) constructs locally optimal schedules for the special case of this problem where there are only two job types. Gupta (1988) also describes a heuristic for the more general problem with an arbitrary number of job classes. Mason and Anderson (1991) present properties of the optimal solution for this problem, and suggest an efficient branch-and-bound solution approach. Scheduling problems with job type dependent setup times have also been studied in parallel processing environments (see, *e.g.*, Wittrock 1986, Bitran and Gilbert 1990, and Tang 1990).

Two key aspects of the problem considered in this paper distinguish it from the research described above. First, while previous models have assumed a given set of work that must be processed by the facility, we combine the selection of a subset of the available jobs with decisions concerning how the accepted work should be scheduled. In addition, we utilize a general profit maximization objective rather than the traditional flow time and tardiness-related criteria found in the literature. While jobs of the same class can conceivably be processed in more than one batch in the schedule that maximizes total profit, for simplicity we restrict attention to solutions in which accepted jobs of the same class are processed in single batches (see Webster and Baker 1995 for a discussion of structural properties of the optimal schedule for a given set of jobs when batch splitting is allowed).

Consider a special case of the problem in which all available work facing the facility is of a single job class k, *i.e.*, $J = J_k = \{1, 2, ..., n_k\}$ and $n = |J_k| = n_k$. Associated with each job j is processing time t_j, delay penalty a_j, and revenue function $R_j(C_j)$ given by (1). The objective is to determine which jobs should be accepted for processing, and the sequence in which accepted jobs should be ordered, to maximize the profit realized by the facility. The following result allows the optimal sequence corresponding to a given set of accepted jobs to be immediately specified.

Proposition 1. There exists an optimal schedule in which the set of accepted jobs of the same class are sequenced in nondecreasing order of the ratio t_j/a_j, *i.e.*, if $t_i/a_i \leq t_j/a_j$ for accepted jobs $i, j \in J_k$, then job i precedes job j in at least one optimal schedule.

All proofs are provided in the Appendix.

Proposition 1 is extremely useful in formulating the single job class problem with an objective of maximizing total profit. Suppose job class k commences processing at time S_k (for the case of a single job class, S_k represents the setup time for job class k; for the case of multiple job classes, S_k represents the total time allocated to job classes sequenced prior to k), and is allocated T_k time units of the facility's capacity. For each $j \in J_k$, let x_j be defined as follows:

$$x_j = \begin{cases} 1, & \text{if job } j \text{ is accepted for processing} \\ 0, & \text{otherwise.} \end{cases}$$

Without loss of generality, we can assume that the jobs in J_k are numbered in nondecreasing order of the ratio t_j/a_j. The single job class problem (STP) can then be formulated as follows:

$$\text{(STP)} \qquad F_k(S_k, T_k) = \max \sum_{j \in J_k} \left[R_j^0 - a_j \left(S_k + \sum_{i=1}^{j} t_i x_i \right) \right] x_j \tag{2}$$

$$\text{s.t.} \qquad \sum_{j \in J_k} t_j x_j \leq T_k \tag{3}$$

$$x_j \in \{0, 1\}, \quad j \in J_k. \tag{4}$$

Formulation (STP) can be easily linearized by defining decision variable $z_{ij} = x_i x_j$.

(LSTP)
$$F_k(S_k,T_k) = \max\left[\sum_{j\in J_k}\left(R_j^0 - a_jS_k\right)x_j - \sum_{j\in J_k}\sum_{i=1}^{j} a_jt_iz_{ij}\right] \tag{5}$$

s.t. (3), (4) and

$$x_i + x_j - 1 \le z_{ij}\ , \quad i \in J_k,\ j \in J_k \tag{6}$$

$$0 \le z_{ij} \le 1\ , \quad i \in J_k,\ j \in J_k. \tag{7}$$

Constraints (6) ensure that the values of variables x_i, x_j, and z_{ij} are consistent with the definition of z_{ij}. Observe that variable z_{ij} assumes a value of 1 if and only if $x_i = 1$ and $x_j = 1$; otherwise, constraint (6) is redundant, and the form of the objective function guarantees that z_{ij} takes on its lowest possible value, $z_{ij} = 0$.

The complexity of problem (STP) is easily established by noting that the special case in which $a_j = 0$ for all $j \in J_k$ is a knapsack problem (this structure is particularly clear in formulation (LSTP)), which is known to be NP-hard (Garey and Johnson 1979). However, the problem is efficiently solved for reasonably sized problems using dynamic programming (see Section 3.1). Though the mixed integer linear programming formulation (LSTP) suggests a Benders Decomposition approach (see Benders 1962), computational concerns, such as slow convergence and the requirement that problem (STP) be repeatedly solved in the solution of the multiple job class problem, favored a dynamic programming approach.

Building on problem (STP), we can formulate the multiple job class problem with an objective of maximizing total profit. Let:

$$Y_{kr} = \begin{cases} 1, & \text{if job class } k \text{ is the } r^{th} \text{ batch processed in sequence} \\ 0, & \text{otherwise,} \end{cases}$$

and T_{kr} the amount of time allocated to job class k when that batch is processed r^{th} in sequence. $T_{kr} = 0$ for $r = 1, 2, ..., m$ indicates that no jobs of class k are accepted for processing in the facility. The multiple job class problem can be formulated as follows:

$$\text{(MTP)} \qquad f(M,T) = \max \sum_{k=1}^{m} \sum_{r=1}^{m} F_k \left(\sum_{\ell=1}^{r-1} \sum_{\substack{k'=1 \\ k' \neq k}}^{m} T_{k'\ell} Y_{k'\ell} \, , \, T_{kr} \right) Y_{kr} \tag{8}$$

$$\text{s.t.} \qquad \sum_{k=1}^{m} \sum_{r=1}^{m} T_{kr} Y_{kr} \le T \tag{9}$$

$$\sum_{k=1}^{m} Y_{kr} = 1 \, , \quad r = 1, 2, ..., m \tag{10}$$

$$\sum_{r=1}^{m} Y_{kr} = 1 \, , \quad k = 1, 2, ..., m \tag{11}$$

$$0 \le T_{kr} \le T \, , \quad k = 1, 2, ..., m, \; r = 1, 2, ..., m \tag{12}$$

$$Y_{kr} \in \{0, 1\} \, , \quad k = 1, 2, ..., m, \; r = 1, 2, ..., m. \tag{13}$$

The objective (8) of problem (MTP) is to maximize total profit for the facility, where function $F(\cdot,\cdot)$ is as defined for problem (STP). Constraint (9) guarantees that the amount of time allocated for processing job classes does not exceed the total capacity (T) of the facility, and constraints (10) and (11) ensure that each job class is assigned to exactly one position in the processing sequence.

Problem (MTP) is clearly NP-hard, since the special case of a single job class, *i.e.*, problem (STP), is NP-hard. In the next section, we present dynamic programming algorithms for obtaining solutions to problems (STP) and (MTP).

3. Solution Approaches

We first develop a dynamic programming algorithm for the single job class problem (STP) in Section 3.1. In the process, we characterize properties of the recursion which, based on a single execution of the algorithm, allow values of the function $F_k(S_k, T_k)$ to be easily calculated for any combination of parameters S_k and T_k. Capitalizing on these properties, we propose a computationally efficient dynamic programming algorithm for the multiple job class problem (MTP) in Section 3.2.

3.1. A Dynamic Programming Algorithm for Problem (STP)

We again consider a single job class k and the set of jobs $J_k = \{1, 2, ..., n_k\}$ corresponding to job class k. The objective is to determine which of the available jobs to accept for processing, and the order in which the accepted jobs should be sequenced to maximize total profit.

Let $G_s^k(S_k, \mu)$ denote the maximum profit earned by processing at most s jobs of type k $(0 \le s \le n_k)$, with the first job processed at time S_k, and a total of μ time units allocated for processing job class k. We can write the following recursive relationship:

$$G_0^k(S_k, \mu) \equiv 0\ , \quad \text{for any } 0 \le \mu \le T_k,$$

$$G_s^k(S_k, \mu) = \max_{\substack{j \in J_k, \\ \tau : \tau + t_j \le \mu}} \left\{ G_{s-1}^k(S_k, \tau) + R_j^0 - a_j(\tau + t_j) \right\}\ , \quad \text{for } 1 \le s \le n_k, 0 \le \mu \le T_k. \tag{14}$$

Expression (14) is a forward DP recursion that terminates after stage n_k, yielding:

$$F_k(S_k, T_k) = G_{n_k}^k(S_k, T_k). \tag{15}$$

Since we assume that jobs of class k are numbered in nondecreasing order of the ratio t_j/a_j, the optimal sequence $\sigma_k(S_k, T_k)$ consists of the set of accepted jobs obtained from (14) arranged in numerical order.

While the number of stages in the recursion represented by expression (14) is finite and equal to the number of jobs in J_k, the associated state space is potentially infinite. However, we show below that only a finite number of discrete values of the state variable μ need to be considered in determining the optimal schedule. For any subset $Q_s \subset J_k$ such that $|Q_s| = s$, let p_{Q_s} refer to the sum of the processing times for those jobs contained in Q_s, or:

$$p_{Q_s} = \sum_{j \in Q_s} t_j. \tag{16}$$

Taken over all possible subsets of $Q_s \subset J_k$ containing exactly s jobs, we obtain the set of processing time sums for subsets of size s, denoted PS-s. The following result indicates that the state space associated with function $G_s^k(S_k, \mu)$ is made up of the processing time sums contained in sets PS-s,PS-$(s-1)$,...,PS-(1).

Proposition 2. $G_s^k(S_k, \mu)$ is a piecewise constant function, nondecreasing in μ and right-hand continuous, with breakpoint values of μ contained in the set of processing time sums for subsets of at most size s.

According to Proposition 2, at stage s of the dynamic programming recursion the state space consists of at most $\sum_{\ell=1}^{s} \binom{n_k}{\ell}$ possible values for the state variable μ. Thus, for a given value of S_k, at most 2^{n_k} distinct subsets of the n_k jobs must be evaluated to determine the optimal policy.

The example contained in Table 1, which depicts a single class of jobs available for processing, illustrates the solution process by providing profit calculations (assuming no setup time for the job class) for all possible subsets of the four jobs. A graph of the maximum associated profit as a function of the total amount of capacity allocated to job class k at time 0 is shown in Figure 1. Observe in Table 1 that the profit associated with any subset Q_s can alternatively be obtained from the profit of some subset Q_{s-1}. Similarly, the impact of delaying the processing of any subset by S_k time units can be directly evaluated from its base profit, *i.e.*, the profit when $S_k = 0$. For example, the base profits of subsets $\{1,3\}$ and $\{2,4\}$ are given in Table 1 as 61.2 and 61.0, respectively; therefore, if job class k commences processing at time 0 and a total of 6 units of capacity are available, jobs 1 and 3 should be accepted and processed in numerical order. However, delaying the processing of job class k by S_k time units decreases the profitability of any subset Q_s by an amount equal to $S_k \sum_{j \in Q_s} a_j$; hence, if $S_k = 3$ the profits of subsets $\{1,3\}$ and $\{2,4\}$ are $61.2 - 0.8(3) = 58.8$ and $61.0 - 0.7(3) = 58.9$, respectively. Thus, subset $\{2,4\}$ now yields a higher profit. In general, both the base profit of a subset (*i.e.*, the profit at $S_k = 0$) and the rate at which that profit decreases as a function of processing delays (as captured by the quantity $S_k \sum_{j \in Q_s} a_j$) must be considered in determining the optimal policy for accepting and scheduling jobs. This observation significantly speeds profit calculations, thus contributing to computational efficiency when problem (STP) must be repeatedly solved over a range of values for S_k, as in the solution procedure for problem (MTP) described in Section 3.2.

Figure 1: Maximum Profit for Various Levels of Capacity (Single Job Class Example)

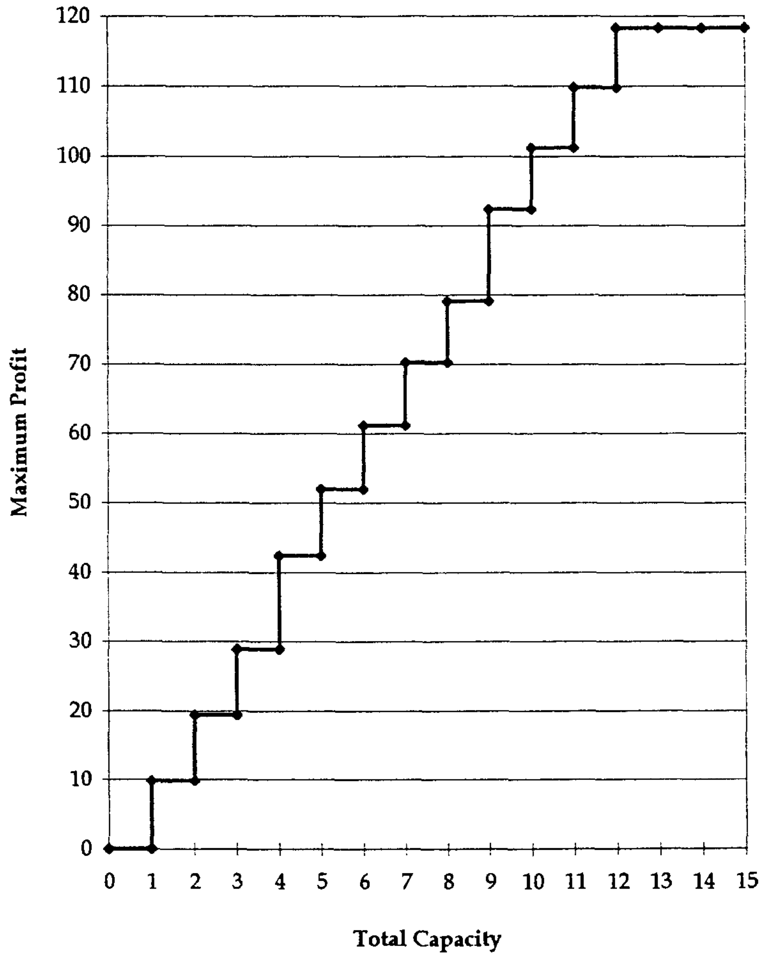


Table 1: Single Job Class Example

Job(j)	t_j	a_j	R_j^0	$\frac{t_j}{a_j}$
1	1	0.2	10	5.00
2	2	0.3	20	6.67
3	5	0.6	55	8.33
4	4	0.4	44	10.00

Sequence	$\mu = \sum_{j \in Q_s} t_j$	Profit Calculation	Alternative Profit Calculation
1	1	$10 - .2(1) = 9.8^*$	–
2	2	$20 - .3(2) = 19.4^*$	–
3	5	$55 - .6(5) = 52.0^*$	–
4	4	$44 - .4(4) = 42.4^*$	–
1–2	3	$30 - .2(1) - .3(3) = 28.9^*$	$G_1^k(0,1) + 20 - .3(3) = 28.9$
1–3	6	$65 - .2(1) - .6(6) = 61.2^*$	$G_1^k(0,1) + 55 - .6(6) = 61.2$
1–4	5	$54 - .2(1) - .4(5) = 51.8$	$G_1^k(0,1) + 44 - .4(5) = 51.8$
2–3	7	$75 - .3(2) - .6(7) = 70.2^*$	$G_1^k(0,2) + 55 - .6(7) = 70.2$
2–4	6	$64 - .3(2) - .4(6) = 61.0$	$G_1^k(0,2) + 44 - .4(6) = 61.0$
3–4	9	$99 - .6(5) - .4(9) = 92.4^*$	$G_1^k(0,5) + 44 - .4(9) = 92.4$
1–2–3	8	$85 - .2(1) - .3(3) - .6(8) = 79.1^*$	$G_2^k(0,3) + 55 - .6(8) = 79.1$
1–2–4	7	$74 - .2(1) - .3(3) - .4(7) = 70.1$	$G_2^k(0,3) + 44 - .4(7) = 70.1$
1–3–4	10	$109 - .2(1) - .6(6) - .4(10) = 101.2^*$	$G_2^k(0,6) + 44 - .4(10) = 101.2$
2–3–4	11	$119 - .3(2) - .6(7) - .4(11) = 109.8^*$	$G_2^k(0,7) + 44 - .4(11) = 109.8$
1–2–3–4	12	$129 - .2(1) - .3(3) - .6(8) - .4(12) = 118.3^*$	$G_3^k(0,8) + 44 - .4(12) = 118.3$

$*$ Optimal profit associated with breakpoint value of μ.

Let B_s^k represent the set of breakpoint values of μ associated with the function $G_s^k(S_k, \mu)$, with $0 \le \mu \le T_k$. The following results are useful in reducing the number of subsets requiring explicit evaluation in the search for the optimal single job class schedule.

Lemma 1. For $2 \le s \le n_k$, $B_s^k \subset P_s = \left\{p_r : p_r = p_\ell + t_j, p_\ell \in B_{s-1}^k, j \in J_k, \text{ and } p_r \le T_k\right\}$.

Lemma 2. If $G_{s+1}^k(S_k, T_k) = G_s^k(S_k, T_k)$ for some $1 \le s \le n_k - 1$ and B_{s+1}^k contains only processing time sums of at most size s, then $F_k(S_k, T_k) = G_s^k(S_k, T_k)$.

Lemma 1 indicates that in determining the set of processing time sums that must be considered at stage $s+1$ of the solution process, attention can be confined to direct augmentations of the set of breakpoint values of μ associated with the immediately previous stage. Lemma 2 provides a test for terminating the dynamic programming procedure prior to stage n_k. These results are instrumental in achieving high computational efficiency in the implementation of the (STP) solution procedure. Our computational experience with the algorithm indicates that single job class problems involving 10 jobs can be solved in approximately 0.5 CPU seconds on an IBM 4381.

3.2. A Dynamic Programming Algorithm for Problem (MTP)

We now consider the situation where the set of available work consists of m different types of jobs. We assume that the optimal profit $F_k(S_k, T_k)$ and associated optimal sequence of accepted jobs $\sigma_k(S_k, T_k)$ can be readily obtained for any combination of S_k and T_k by solving the single job class problem for each job type k. According to Proposition 1, we may also assume that jobs of a given job class are numbered in nondecreasing order of the ratio t_j/a_j.

Let $V_q(L,t)$ denote the maximum profit earned if at most q types of jobs from the set $L \subset M$ are processed for a total of t time units starting at time 0. Recall that h_k represents the setup time incurred prior to the production of a batch of job class k. We can then write the following recursive relationship for $0 \le t \le T$ and $L \subset M$:

$$V_1(L,t) = \max_{k \in L} \{F_k(0,t)\} \ , \quad \text{for } L \neq \emptyset,$$

$$V_q(L,t) = \max_{\substack{\tau, k \in L: \\ \tau + h_k + \min_{j \in J_k}(t_j) \le t}} \{V_{q-1}(L \backslash \{k\}, \tau) + F_k(\tau + h_k, t - \tau - h_k)\} \ , \quad \text{for } |L| \ge q. \tag{17}$$

Expression (17) is also a forward DP recursion that terminates after stage m, yielding:

$$f(M,T) = V_m(M,T). \tag{18}$$

Again observe that while the state space of the recursion represented by expression (17) is infinite, only a finite number of states need to be considered in determining the value $f(M,T)$. In support of this conclusion, note that at every stage the only sets L that must be considered are M and $M \setminus \{k\}$. Let $B'(f)$ represent the set of breakpoint values of t of the piecewise constant function f over its domain. For notational consistency, we require $B'\left(G_s^k(S_k, \mu)\right) \equiv B_s^k$. The finiteness of the state space is established by the following result.

Lemma 3. $B'(V_q(L,t)) \subset \bigcup_{k \in L} B'(F_k(0,t))$.

To illustrate the solution process for problems involving multiple job classes, consider the 2-job type example presented in Table 2. We assume for simplicity that no setup time is required for either job class, *i.e.*, $h_1 = h_2 = 0$, and that the total amount of available capacity is 9 time units. The dynamic programming algorithm first determines the policy yielding the maximum profit for a given level of capacity by assuming that only jobs from a single job class are accepted for processing. Figure 2 indicates that the optimal profit $V_1(\{1,2\},t)$ is determined for $0 \leq t \leq T$ by comparing the optimal profits $F_1(0,t)$ and $F_2(0,t)$ for each job class considered individually. Thus, Figure 2 shows that a policy that accepts and processes only jobs of type 1 yields maximum profits for $t < 3$ and $t \geq 9$, and that a schedule constructed from only jobs of type 2 is optimal for $3 \leq t < 9$.

Policies in which jobs from both classes are accepted and processed must next be considered. Assuming first that jobs of class 1 will be processed first, the amount of time allocated to the production of class 1 can be varied over the feasible range, with the remainder of the available capacity dedicated to the production of jobs of type 2, as illustrated at the bottom of Table 2. Also shown are the profit implications of producing class 2 jobs first in sequence over all feasible allocations of capacity. We conclude from the results in Table 2 that scheduling job 3 of type 1 first, followed by job 3 of type 2, yields maximum profit 102.4.

Similar problems involving multiple jobs from several job classes can be efficiently solved using the (MTP) model. Our computational experience indicates that problems made up of as many as 5 job classes, 10 jobs per class, can be solved in approximately 60 CPU seconds on an IBM 4381.

Figure 2: Maximum Profit for Various Levels of Capacity
(Two Job Class Example)

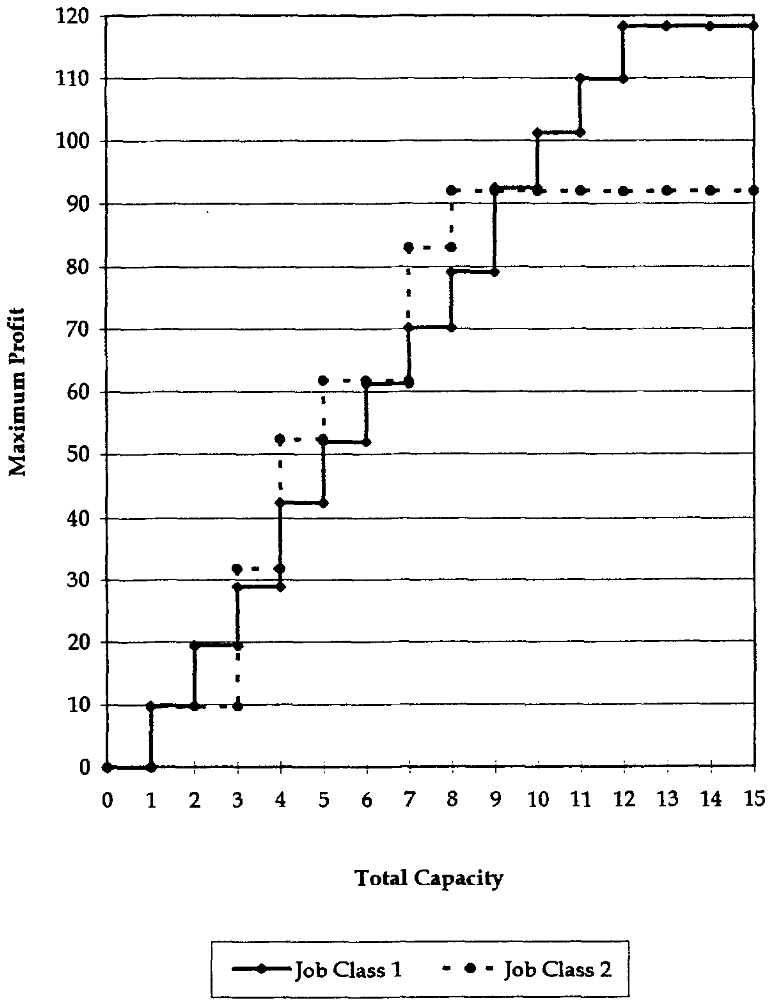


Table 2: Two Job Class Example

Job Class 1					Job Class 2				
Job(j)	t_j	a_j	R_j^0	$\frac{t_j}{a_j}$	Job(j)	t_j	a_j	R_j^0	$\frac{t_j}{a_j}$
1	1	0.2	10	5.00	1	1	0.3	10	3.33
2	2	0.3	20	6.67	2	3	0.4	33	7.50
3	5	0.6	55	8.33	3	4	0.4	54	10.00
4	4	0.4	44	10.00					

Sequence of Job Classes = 1–2

Capacity Allocated to Job Class 1	Job Class 1 Profit	Capacity Allocated Job Class 2	Job Class 2 Profit	Total Profit
1	9.8	8	90.7	100.5
2	19.4	7	81.4	100.8
3	28.9	6	59.6	88.5
4	42.4	5	58.9	101.3
5	52.0	4	50.4	102.4
6	61.2	3	29.4	90.6
7	70.2	2	7.6	77.8
8	79.1	1	7.3	86.4

Sequence of Job Classes = 2–1

Capacity Allocated to Job Class 2	Job Class 2 Profit	Capacity Allocated Job Class 1	Job Class 1 Profit	Total Profit
1	9.7	8	78.0	87.7
3	31.8	6	58.9	90.7
4	52.4	5	49.6	102.0
5	61.7	4	40.4	102.1
7	83.0	2	17.3	100.3
8	91.9	1	8.2	100.1

4. Marketing/Manufacturing Interface Issues

In attempting to maximize the local performance of their individual functions, marketing and manufacturing domains within the firm often fail to optimize overall system performance (see, *e.g.*, Shapiro 1977, Montgomery and Hausman 1986, Karmarkar and Lele 1989 and Eliashberg and Steinberg 1991 for extensive discussion on sources of conflict between marketing and manufacturing). Recent research has yielded formal models that capture important elements of the marketing/manufacturing interface, explain where and why conflicts between these two functional areas arise, and suggest how coordinating policies that are beneficial to the firm as a whole can be developed and implemented (see, *e.g.*, the review paper of Eliashberg and Steinberg 1991). In this section, we discuss how a model for accepting and scheduling jobs from multiple classes can be used both as a conceptual tool for demonstrating to functional managers the suboptimality of functionally driven performance measures, and as an analysis tool for focusing the cross-functional coordination efforts of managers.

We first emphasize the suboptimality of functionally-driven performance measures in our modeling environment. Consider the example presented in Table 3, where six jobs of a single class k are available for processing by a facility with finite capacity $T = 20$. Assume no setup time for class k, *i.e.*, $h_k = 0$.

Table 3

Job(j)	t_j	a_j	R_j^0	$\frac{t_j}{a_j}$	$\frac{R_j^0 - a_j t_j}{t_j}$	$t_j \times a_j$
1	11	2.0	66	5.5	4.00	22.0
2	9	1.5	48	6.0	3.83	12.5
3	3	0.4	12	7.5	3.60	1.2
4	4	0.5	17	8.0	3.75	2.0
5	7	0.7	30	10.0	3.59	4.9
6	6	0.3	24	20.0	3.70	1.8

A marketing perspective focuses on the revenue implications of accepting individual jobs while ignoring detailed operational constraints such as setup times, job-specific delay penalties, and capacity limitations. A quantity that supports a revenue maximization objective is $\frac{R_j^0 - a_j t_j}{t_j}$, calculated for each job in Table 3, which indicates the revenue (net of the minimum cost that must be incurred if job j is processed immediately) per unit of consumed capacity generated by including job j in the portfolio of accepted work. Using this ratio as a greedy measure for accepting jobs for processing results in the selection of jobs 1 and 2 which, when processed in numerical order, yield a total profit of:

$$\text{Profit} = (66 + 48) - 2.0(11) - 1.5(20) = 62.$$

From a traditional operations perspective, minimizing manufacturing costs while efficiently utilizing available capacity is of primary importance. Quantities that support a cost minimization objective are the ratio t_j/a_j, which is the inverse of the cost per unit of consumed capacity, and the product $t_j \times a_j$, the minimum cost that must be incurred if job j is processed immediately. Each of these quantities is also calculated for each job in Table 3. Using either measure in a greedy procedure for accepting jobs results in selection of jobs 3, 4, 5, and 6 which, when optimally sequenced, yield a total profit of:

$$\text{Profit} = (12 + 17 + 30 + 24) - 0.4(3) - 0.5(7) - 0.7(14) - 0.3(20) = 62.5.$$

The formulation of problem (STP) focuses simultaneously on revenue and cost concerns, while accounting for sequencing and capacity restrictions. The dynamic programming algorithm for problem (STP) accepts jobs 1, 3, and 6, which implies a total profit of:

$$\text{Profit} = (66 + 12 + 24) - 2.0(11) - 0.4(14) - 0.3(20) = 68.4,$$

or roughly a 10% improvement over the solutions generated by the individual functional areas. Note that this simple example does not penalize the functionally motivated heuristics for either capacity underutilization (all solutions fully consume available capacity) or setup delays (the example assumes a single job class and no setup times). To illustrate how both of these elements can affect the performance of the greedy heuristics, consider a slight variation on the example presented in Table 3. Suppose now that jobs 1, 3, and 6 belong to job class 1 and jobs 2, 4, and 5 to job class 2, and that a setup time of 1 time unit is incurred each time production is switched from one job type to another, *i.e*, $h_1 = h_2 = 1$. The capacity of the facility remains $T = 20$. Using revenue per unit of consumed capacity to evaluate and select individual jobs and ignoring setup and capacity restrictions, the marketing heuristic again attempts to accept jobs 1 and 2, whose setup and processing requirements exceed available capacity by 2 time units. Thus, either an alternative source of processing capacity (*e.g.*, overtime or subcontracting) must be utilized, or one of jobs 1 and 2 must be rejected and replaced by other available work (a pure greedy approach would reject job 2 and ultimately accept jobs 3 and 4, for a total profit of 55). The manufacturing heuristic again attempts to accept jobs 3 through 6, violating the capacity of the facility by at least 2 time units. If alternative capacity cannot be acquired, one or more of these jobs must be rejected (a pure greedy approach would reject job 5, yielding a total profit of 42.4). The optimal solution is to accept and process in numerical order jobs 1 and 6, resulting in the maximum profit of 60.6.

These examples highlight the impact of exclusive focus on functional performance, and provide insight on how detailed operational models can be used to rationalize and communicate when and why the interests of each functional area must be compromised to benefit the firm as a whole. To illustrate how our scheduling model can be used to coordinate marketing/manufacturing policies, consider again the 6-job example given in Table 3, and assume that jobs 1, 3, and 6 belong to job class 1 and jobs 2, 4, and 5 to job class 2 (for simplicity, we assume no setup time for either job class). We adopt the following interpretation for the revenue function $R_j(C_j)$ given in (1): let R_j^0 represent the price quoted to a customer for job j (assume that the marketing area makes this pricing decision), and a_jC_j reflects the cost of holding job j until its completion at time C_j (this cost is clearly affected by decisions concerning which jobs to accept and how accepted work is scheduled). For the problem instance given in Table 3, the optimal solution is to accept only jobs of class 1, specifically jobs 1, 3, and 6. If the mix of jobs in this example are representative of the demands typically faced by the firm, then such a solution could have long run marketing implications, since consistent rejection of type 2 jobs sends a strong signal to the market about the firm's processing capabilities. A potentially relevant long-term concern for a firm in this position is to determine appropriate marketing and manufacturing actions to ensure that a mix of jobs from both job classes may be profitably processed by the facility.

The marketing function, with its pricing authority and market information, may find the detailed scheduling model (MTP) useful in exploring the sensitivity of job acceptance decisions to alternative pricing policies. Table 4 illustrates the effect of price changes for jobs 1 and 2 on the optimal set of accepted jobs. Similar tables can be generated for other combinations of jobs.

Table 4

R_1^0	R_2^0	Job Types Accepted	Optimal Sequence	Maximum Profit
66	48	1	1–3–6	68.4
66	49-53	1	1–3–6	68.4
66	54	1 and 2	2–4–6	69.3
67	54	1	1–3–6	69.4
67	55	1 and 2	2–4–6	70.3
68	55	1 and 2	1–2	71.0

Table 4 indicates that, all else remaining equal, the price of job 2 must be increased substantially for the firm to profitably include it in its portfolio of accepted work. Table 4 also emphasizes how sensitive the processing mix is to the relative price of available jobs. Finally, in order for the marketing heuristic of selecting jobs for processing according to their revenue per unit of consumed capacity to correspond to the optimal solution, the prices of both jobs 1 and 2 must be increased from their current level.

The manufacturing function, with its control over cost reduction alternatives for the facility, can employ the detailed scheduling model to explore how changes in the parameter a_j

can profitably move the firm towards an operating mix that includes both job classes. Table 5 illustrates the effect of cost changes for jobs 1 and 2 on the optimal set of accepted jobs.

Table 5

a_1	a_2	Job Types Accepted	Optimal Sequence	Maximum Profit
2.0	1.9	1	1–3–6	68.4
1.9	1.5	1	1–3–6	69.5
2.0	1.2-1.4	1	1–3–6	69.5
2.0	1.1	1 and 2	1–2	70.0

Table 5 clearly indicates that in order to achieve the desired operating mix, cost reduction efforts must be centered on job 2.

These simple examples illustrate the value of a detailed model in structuring cross-functional debate, and in developing integrated functional policies for meeting the objectives of the entire firm.

5. Manufacturing Focus

The manufacturing strategy literature has consistently stressed the importance of manufacturing focus, *i.e.*, dedicated effort towards achieving outstanding operational performance along a narrowly defined set of measures, as a means of establishing a firm's operations as a sustainable competitive asset (see, *e.g.*, Skinner 1969, 1974, Hayes and Wheelwright 1984, and Hill 1989). While the conceptual importance of manufacturing focus is clear, operations managers must realize that focusing an actual facility implies operational trade-offs that should be quantified and structured as much as possible before such a policy is implemented. In particular, issues concerning how certain elements of the specific operational environment (*e.g.*, product volumes, product mix, setup times, and system capacity) affect the type of focusing strategy that can be profitably applied to a given facility must be carefully considered. Conceptual manufacturing strategy models (see, *e.g.*, Skinner 1974, Hill and Duke-Wooley 1983, Hill 1989, and Berry *et al.* 1991) fail to address such detailed questions. Detailed models, such as the scheduling models presented in this paper, provide valuable insight into how the concept of manufacturing focus can be operationalized for a specific setting, and how the associated operational trade-offs can be structured and quantified as an aid for managerial decision making. The importance of operational model building for addressing issues concerning manufacturing focus is illustrated by the examples presented in this section.

First consider a manufacturing environment that might reasonably adopt a product focused strategy, *i.e.*, a conscious policy of accepting and producing only a single type of job. The example presented in Table 6 involves 3 job classes. Job class 1 is characteristic of a high-volume, standardized good that enjoys relatively large demand (both in terms of the number of available jobs and total processing time), but low profit per unit of consumed capacity. Job classes 2 and 3 represent low-volume, customized products characterized by relatively low demand and high unit profits. Suppose that a constant setup time of h time units is incurred each time the facility changes over from the production of one job class to another (*i.e.*, $h_1 = h_2 = h_3 = h$), and that the total amount of capacity available to the facility is 28 time units.

Table 6

Job Class 1				Job Class 2			
Jobs(j)	t_j	a_j	R_j^0	Jobs(j)	t_j	a_j	R_j^0
1	7	1.50	60	1	5	1.60	65
2	2	0.40	20	2	1	0.15	30
3	6	0.80	50	3	8	0.20	70
4	3	0.30	55				
5	9	0.50	80	Job Class 3			
				1	4	0.70	45
				2	10	0.60	100

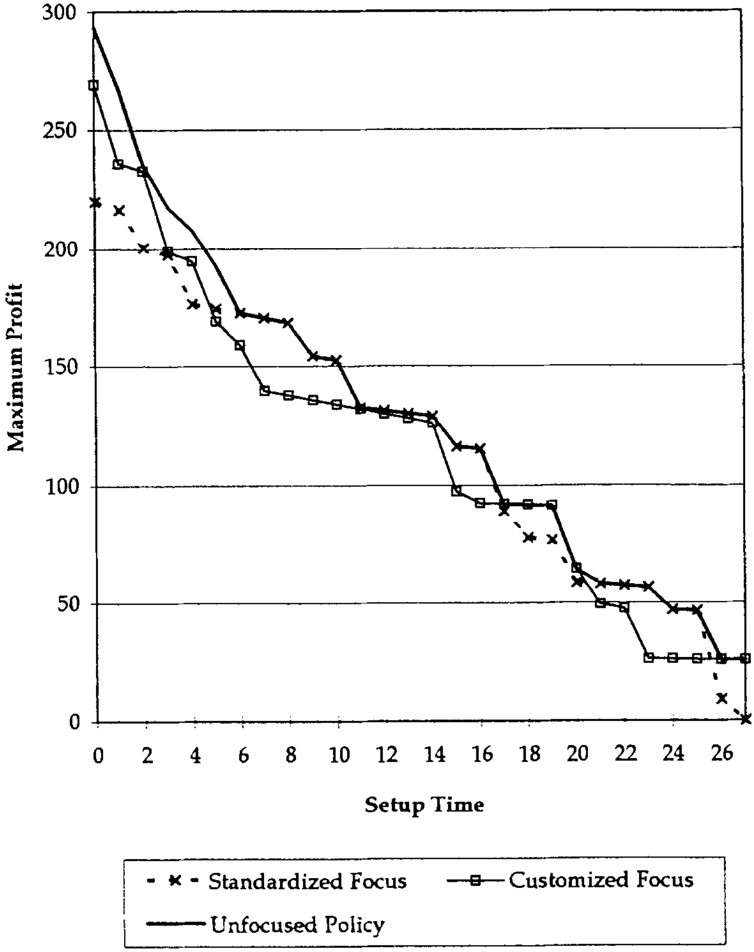
For the operational environment described by the data in Table 6, we can use the (STP) and (MTP) models to determine how the profitability of a focused strategy (centered on either high volume/low profit or low volume/high profit job classes) is affected by environmental factors such as setup times. Given setup time h, the (STP) model is applied to job class 1 to obtain the optimal solution for a strategy targeting the standardized product. Similarly, the (MTP) model is applied to job classes 2 and 3 to determine the optimal solution when only the set of customized jobs is considered. Finally, the (MTP) model is used to solve the 3-job type problem to obtain the best unfocused policy. The results for a wide range of setup times are graphically depicted in Figure 3.

Figure 3 shows that a focused policy is optimal only for relatively high levels of setup time ($h \geq 6$, or 21% of total capacity). We note that the optimal unfocused solution accepts jobs from all three classes over the range of setup times considered. Interestingly, a specific job j can appear attractive for such a strategy because of a low processing time (t_j), a small delay penalty (a_j), a high initial revenue (R_j^0), or a processing time or job type that fits well with the portfolio of accepted jobs given setup times and a tight capacity constraint.

Over the setup time range $6 \leq h \leq 27$, Figure 3 shows that a focused policy yields the optimal profit. Over most of this range, a strategy targeting the high volume/low profit standardized product is favored. However, a policy focused on low volume/high profit customized job types 2 and 3 is dominant for $17 \leq h \leq 20$. The insight derived from the detailed model, *i.e.*, what setup time levels favor manufacturing focus and when specifically should attention be confined to the set of standardized *vs.* customized products, is virtually impossible to obtain using standard conceptual arguments from the manufacturing strategy literature. A simple graph like Figure 3 can also be used to determine how efforts to reduce setups support a marketing strategy targeting specific market segments, or to assess whether an alternative processing technology allows the firm to compete in new markets.

The recent manufacturing strategy literature has presented several examples of firms that have shifted their strategic emphasis from cost minimization of products targeted for price sensitive markets, to lead time minimization targeted for customers whose demands must be met as quickly as possible (see, *e.g.*, Stalk 1988, Stalk and Hout 1990, Blackburn 1991, and Lindsley *et al.* 1991). This strategic shift provides new opportunities for focusing on market

Figure 3: Profits for Different Focusing Policies Over Various Setup Times



segments characterized by higher unit profits, but also higher penalties for delivery delays. Conceptual strategy models address the general issues relating manufacturing focus and time-based competition; however, detailed models are required to explicitly structure the relevant operational trade-offs, and provide valuable insight into how environmental factors, such as the price premium associated with time competitive products, affect the profitability of focused and unfocused policies.

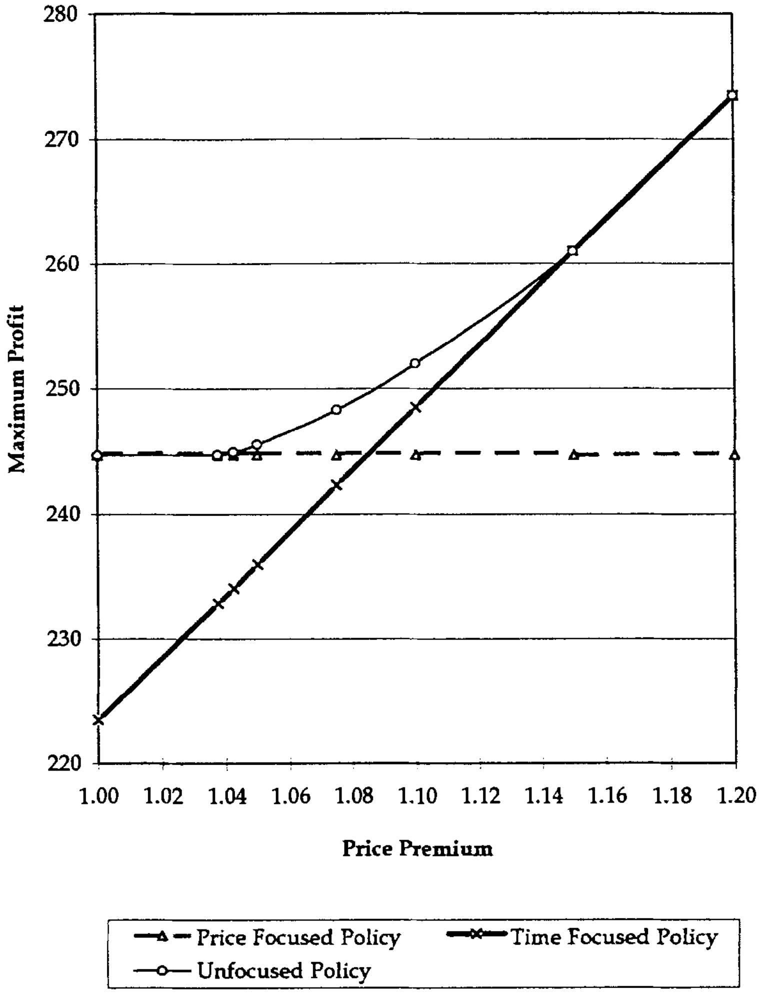
Consider the single job class, 10-job example presented in Table 7. The set of available work consists of two sets of jobs that have identical processing time requirements, but different delay penalties, reflecting demands placed on the facility from both price sensitive and time competitive market segments. Jobs accepted from customers preferring short lead times are charged a premium x over the price of a similar job from the price sensitive market. We are interested in exploring the relationship between the premium x and the profitability of strategies focused respectively on price sensitive and time competitive jobs. Assume that there is no setup time, and that the total amount of capacity available to the facility is 25 time units.

Table 7

Jobs(j)	t_j	a_j	R_j^0
1	1	0.1	10
2	3	0.1	30
3	4	0.1	40
4	7	0.1	70
5	10	0.1	100
6	1	0.5	$10x$
7	3	0.5	$30x$
8	4	0.5	$40x$
9	7	0.5	$70x$
10	10	0.5	$100x$

The (STP) model is applied to jobs 1–5 to determine the optimal schedule when only jobs from the price sensitive market segment are considered for acceptance. Given price premium x, the (STP) model is also applied to jobs 6–10 to obtain the optimal solution for a strategy focused soley on jobs from the time competitive market segment. Finally, the best unfocused policy is identified by using model (STP) to solve the entire 10-job problem. The results for various levels of the price premium x are presented in Figure 4.

Figure 4: Time Competition and Price Sensitive Policies for Various Price Premiums



When the price premium x is low ($x \leq 1.0375$), focusing on price sensitive jobs yields optimal profits that are substantially higher than those obtained from a strategy that accepts only time competitive jobs. For higher levels of x ($x \geq 1.15$), a policy targeting only time competitive jobs is optimal, dominating a price focused strategy by a large and increasing amount. Between these two extremes ($1.0375 \leq x \leq 1.15$), an unfocused policy that accepts both price sensitive and time competitive jobs yields a higher profit than either of the focused alternatives. Detailed analysis such as the above allows specific operational policies to be evaluated in the context of the range of marketing strategies that may be followed concurrently.

In many operational environments, the interactions among factors such as the volume of work available from each job class, job processing times, delay penalties, and initial revenues both within and across job classes complicate the problem of determining an appropriate focused strategy, further justifying the use of detailed operational models to enhance managerial understanding of the capabilities of the production system. To demonstrate this benefit, consider the example problem involving 4 job classes shown in Table 8.

Table 8

Job Class 1

Jobs(j)	t_j	a_j	R_j^0
1	1	0.5	10
2	3	1.0	30
3	2	0.4	20
4	4	0.6	40
5	5	0.7	50
6	7	0.5	65
7	8	0.4	70
8	6	0.2	55
9	10	0.3	85
10	9	0.1	80

Job Class 2

Jobs(j)	t_j	a_j	R_j^0
1	3	1.0	$30x$
2	2	0.4	$20x$
3	7	0.5	$65x$
4	10	0.3	$85x$

Job Class 3

Jobs(j)	t_j	a_j	R_j^0
1	1	0.5	$10x$
2	8	0.4	$70x$
3	9	0.1	$80x$

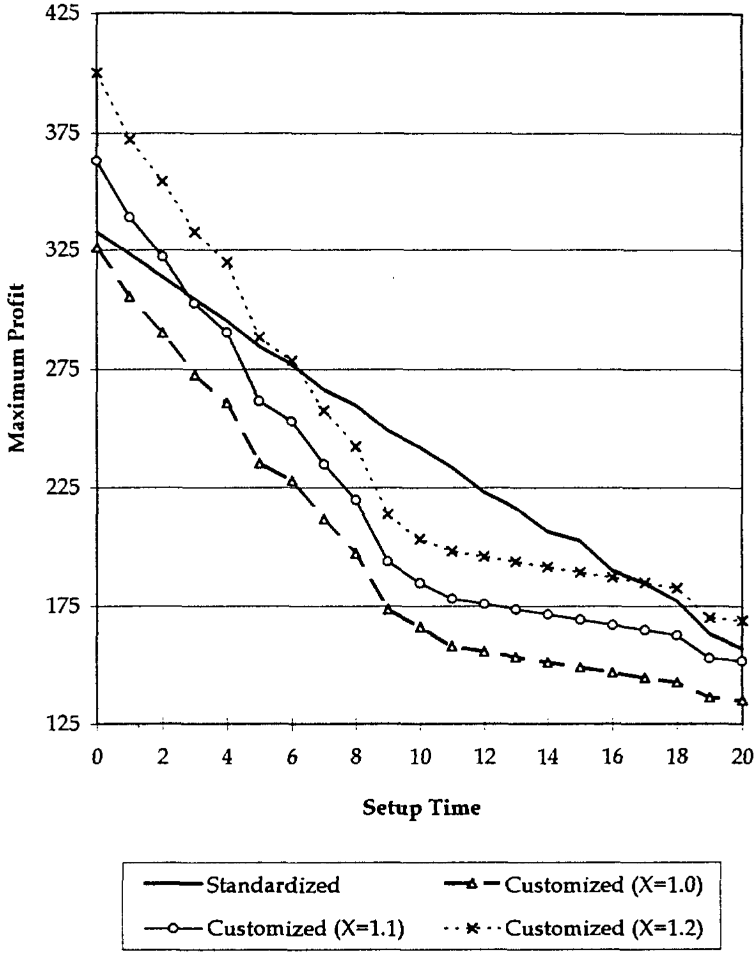
Job Class 4

Jobs(j)	t_j	a_j	R_j^0
1	4	0.6	$40x$
2	5	0.7	$50x$
3	6	0.2	$55x$

Job class 1 represents a set of standardized products that display a wide variety of processing time, delay penalty, and revenue characteristics. Job classes 2–4 represent the customized versions of the products in job class 1; observe also that each customized job realizes a price premium of x over its standardized counterpart. Assume that a constant setup time is incurred each time the facility changes over from the production of one job class to another (*i.e.*, $h_1 = h_2 = h_3 = h_4 = h$). We consider two strategic options. A strategy focused on the standardized products limits the firm to accepting jobs of type 1. We evaluate this option by applying model (STP) to the jobs in class 1. Alternatively, the firm can follow a focused strategy targeting the set of customized products, thus realizing potentially higher profit margins, but at an operational expense of setup downtime that results from processing multiple job classes. This option is evaluated by applying model (MTP) to the jobs in classes 2–4. Models (STP) and (MTP) thus allow the effectiveness of the two strategies to be determined as several key factors affecting the operational environment (setup times, capacity, and price premiums) are varied. The results are summarized in Figure 5 for a facility with 40 time units of capacity, and in Figures 6*a* and 6*b* for lower capacity levels.

The results in Figure 5 indicate that when the price premium realized for customization is low, a strategy focused on the set of standardized products yields higher profits over the entire range of setup times considered. As the price premium is increased, the customization strategy becomes dominant over a small range of low setup times, since at these levels the higher unit profitability of jobs compensates for the lost capacity that results from increasing the number of setups through customization. Interestingly, very high setup time levels, especially when the price premium is also high, also favor a strategy focused on the customized products. This observation stems from the ability of a customization policy to reduce when necessary the required number of expensive setups by only accepting jobs from the most profitable segments of the customized market (in this example, jobs from only 1 of the 3 customized job classes are processed when $h \geq 11$).

Figure 5: Profits for Standardization and Customization Strategies (Capacity = 40)



Figures 6*a* and 6*b* illustrate for this example the impact of decreasing the amount of available capacity on the effectiveness of the standardization and customization strategies. Tighter capacity again favors the customization strategy, due to its ability to focus on the most prof-

Figure 6a: Profits for Standardization and Customization Strategies (Capacity = 20)

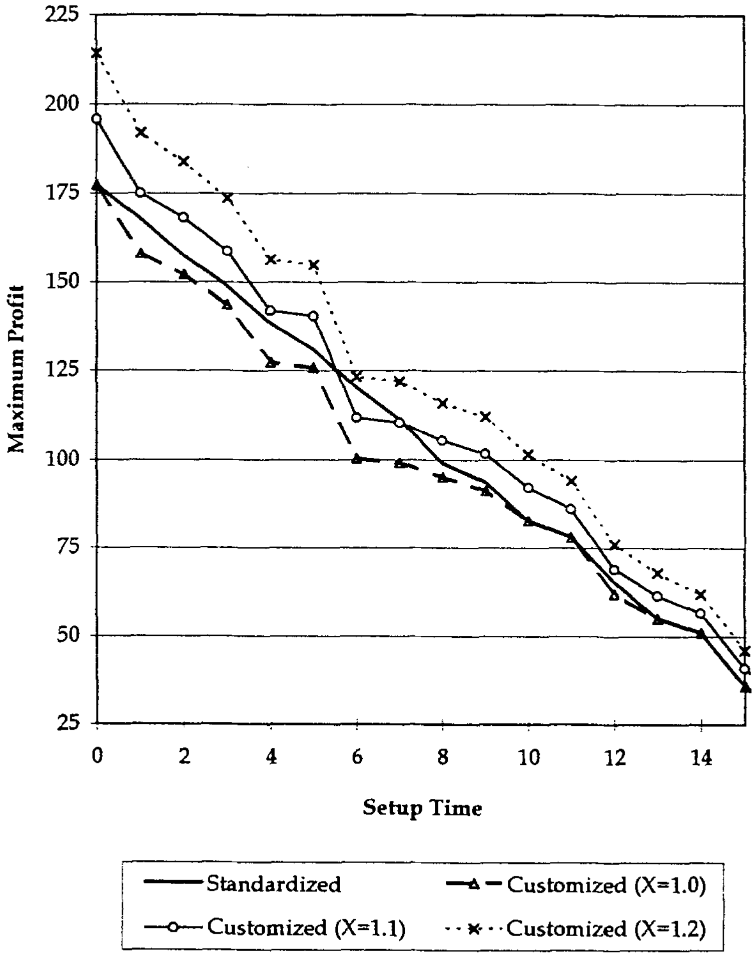
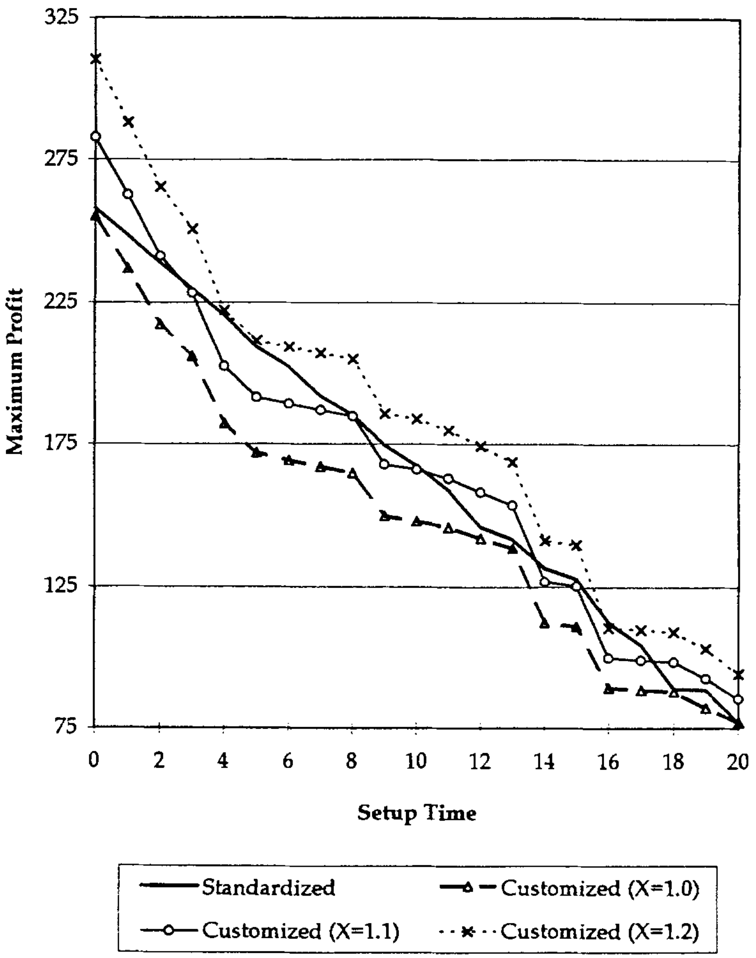


Figure 6b: Profits for Standardization and Customization Strategies (Capacity = 30)



itable segments of the market while maintaining a minimum number of setups. Counter to conventional intuition, which argues that an increase in capacity should favor a customized strategy (since a greater number of setups for customized products can be accomodated), a comparison of Figures 5, 6*a* and 6*b* clearly indicates that capacity expansion options must be evaluated in conjunction with policies for determining the associated levels of setup times and price premiums before concluding that a standardization *vs.* customization policy should be followed.

From an analysis such as that presented in Figure 5, 6*a*, and 6*b*, a firm can evaluate the effectiveness of standardization and customization strategies given current characteristics of the operating environment (*e.g.*, setup times, capacity, and price premiums), and determine what manufacturing efforts (*e.g.*, setup time reduction, alternative sources of capacity) and pricing policies should be followed to profitably move the firm in the desired direction.

6. Conclusions

This paper has considered operating environments in which jobs with varying processing time, delay penalty, and revenue characteristics compete for processing by a single facility. Jobs were partitioned into multiple job classes such that a setup is required whenever jobs from different classes are processed in succession. Given limited processing capacity, the objective was to determine the subset of jobs to accept for processing and the associated order in which the accepted jobs should be sequenced to maximize the total profit realized by the facility. Problem formulations and dynamic programming solution approaches were presented for both the special case where all available work is from a single job class, and the more general case involving multiple job classes. A series of example problems were presented to illustrate how these detailed models could be used to structure and evaluate the operational trade-offs that result from strategic decision making, first focusing on the need to coordinate marketing and manufacturing policies, and finally by considering important issues related to manufacturing focus.

The models presented in this paper reinforce the major role that manufacturing should play in the formulation of corporate strategy. By providing an important link between functional policy decisions and manufacturing performance, detailed models of the operational environment clearly communicate how strategic decisions can either accent or ignore the strengths of the manufacturing system. More effective manufacturing management can also result from the explicit recognition of a corporate-wide objective such as maximizing total profit. Future research should further emphasize the importance of detailed models as a means for determining and highlighting the operational impact of strategic decisions.

Acknowledgments

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Appendix

Proof of Proposition 1. Observe that for any set of accepted jobs $A \subset J_k$ and for any schedule S:

$$\sum_{j\in A} R_j(C_j) = \sum_{j\in A} R_j^0 - \sum_{j\in A} a_j C_j, \qquad (P1-1)$$

where the C_j are determined by schedule S. Since $\sum_{j\in A} R_j^0$ is constant for a given set A, profit is maximized by minimizing $\sum_{j\in A} a_j C_j$, which is accomplished by sequencing the jobs in A in nondecreasing order of the quantity t_j/a_j (see, Smith 1956). ∎

Proof of Proposition 2. Consider two consecutive breakpoints μ_1 and μ_2 as specified in the statement of the proposition. For any $\mu_1 \le \mu \le \mu_2$, if there exists a sequence $\sigma_k(S_k, \mu)$ such that $G_s^k(S_k, \mu) > G_s^k(S_k, \mu_1)$, then the subset of jobs Q appearing in $\sigma_k(S_k, \mu)$ should be such that $p_Q = \sum_{j\in Q} t_j \le \mu_1$. However, this implies that $G_s^k(S_k, \mu_1) \ge G_s^k(S_k, \mu)$, which is a contradiction. Thus, $G_s^k(S_k, \mu) \le G_s^k(S_k, \mu_1)$, and since sequence $\sigma_k(S_k, \mu_1)$ is feasible for $\mu \ge \mu_1$, $G_s^k(S_k, \mu) = G_s^k(S_k, \mu_1)$ for any $\mu_1 \le \mu < \mu_2$.

Observe also that if μ_1 and μ_2 are such that $G_s^k(S_k, \mu_2) > G_s^k(S_k, \mu_1)$, then $G_s^k(S_k, \mu) < G_s^k(S_k, \mu_2)$ for any $\mu_1 \le \mu < \mu_2$, which implies a discontinuity from the left as $\mu \longrightarrow \mu_2$. However, if μ_3 is the breakpoint immediately larger than μ_2, then $G_s^k(S_k, \mu') = G_s^k(S_k, \mu_2)$ for any $\mu_2 \le \mu' < \mu_3$, thus establishing right hand continuity. ∎

Proof of Lemmas 1–3. The details are omitted, since the results follow directly from Proposition 2, the definition of B_s^k, and the recursive relationships (14) (for Lemmas 1 and 2) and (17) (for Lemma 3). ∎

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